International Journal of Engineering Research And Advanced Technology (IJERAT)

DOI: http://dx.doi.org/10.7324/IJERAT.3135

E-ISSN : 2454-6135

Vol.3 (9) Sep -2017

Influence of Fiber Orientation on the Natural Frequencies of Laminated Composite Beams

Imad-Eldin Mahmoud Mahdi[1], Osama Mohammed Elmardi Suleiman[2]

Assistant Professors[1,2]

Department of Mechanical Engineering,

Faculty of Engineering and Technology,

Nile Valley University, Atbara,

River Nile State,

Sudan

ABSTRACT

In this study, the effect of the fiber orientation on the natural frequencies of free vibration of laminated composite beams was investigated. The problem is analyzed and solved using the energy approach which is formulated by a finite element model. In that model, a three-noded element with three degrees of freedom at each node is assumed. Numerical results were obtained for symmetric and non-symmetric laminated beams, and verified by comparisons with other relevant works. Also, some special cases, which related to the title, were studied and presented. The angle of fibers orientation, (θ) *which measured from the longitudinal axis of the beam, is varied from* 0° *to* 90°*. It is found that both symmetrically and anti-symmetrically laminated beams of similar size and end conditions have equal natural frequencies which, generally, decrease as the angle of orientation increases.*

Keywords: *Finite Element Method, First Order Shear Deformation Theory, Free Vibration, Laminated Composites, Natural Frequencies.*

---

1. INTRODUCTION

Composite materials are those formed by combining two or more materials on a macroscopic scale such that they have better engineering properties than the conventional materials, for example, metals. Some of the properties that can be improved by forming a composite material are stiffness, strength, weight reduction, and corrosion resistance, thermal properties, fatigue life, and wear resistance. Most man-made composite materials are made from two materials: a reinforcement material called fiber and a base material, called matrix material.

Laminar composites are those having alternating layers of material bonded together in some manner and include thin coatings, thicker protective surfaces, claddings, bimetallic, laminates, and sandwiches. Laminated composite beams are increasingly being used in many engineering applications in the fields of mechanical and civil engineering, transportation vehicles, marine, aviation and aerospace.

The papers, which are presented here as references, address the problem of the free vibration of laminated composite beams. A theoretical analysis of the vibration of composite beams with solid cross sections was also presented by Teoh and Huang [1], Chandrashekhara et al. [2], Abramovich [3]. In those analyses, the equations of motion were based on a Timoshenko beam model (shear deformation considered). Numerical results showed the effect of the shear deformation and fiber orientation on the natural frequencies. Again, Abramovich and Livshits [4] presented exact solutions for the free vibration of non-symmetrically laminated cross-ply composite beams. Marur and Kant [5] and [6], and McCarthy et al [7] applied higher order shear deformation theories to solve the problem of the free vibration of composite beams.

The first-order shear deformation theory was used by Teboub and Hajela [8] to analyze the free vibration of generally layered composite beams. Hodges et al. [9] presented two different methods, which were simple analytical method and finite element method for the prediction of the natural frequencies and mode shapes of composite beams. In addition to the references mentioned above, references [10], [11], and [12] applied different techniques of the finite element method for the same problem.

The theory used in the present paper comes under the category of displacement theories as classified by Phan and Reddy [13], Osman and Suleiman [14] and [15]. In this theory, which is called first order shear deformation theory (FSDT), the transverse planes, which are originally normal and straight to the middle plane of the beam, are assumed to remain straight but not necessarily normal after normal after deformation, and consequently shear correction factors are employed in this theory to adjust the transverse shear stress, which is constant through thickness.

2. MATHEMATICAL FORMULATION

Some assumptions were made in this analysis, which are: (1) All layers behave elastically; (2) Displacements are small compared with the beam depth; (3) Perfect bonding exists between layers; (4) The laminate is equivalent to a single anisotropic layer; (5) The beam is flat and has a rectangular section and vibrates in a vacuum; and (6) All kinds of damping are neglected. Figure 1 below shows the geometry of a beam drawn in the three dimensions X, Y, and Z or 1, 2, and 3.

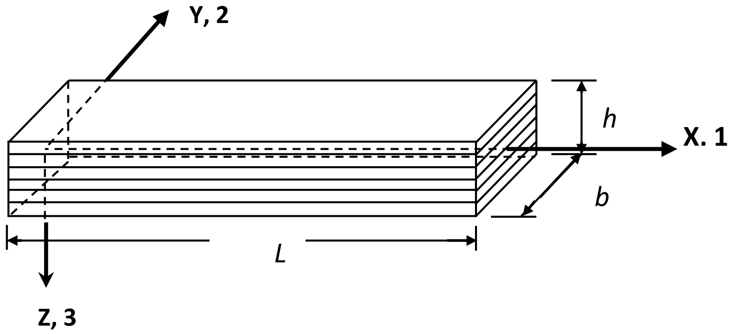


Figure 1 The geometry of a laminated composite beam.

The time-dependent axial and transverse displacements fields are:

$$\left.\begin{aligned} U(x,z,t) &= u(x,t) + z\phi(x,t) \\ W(x,z,t) &= w(x,t) \end{aligned}\right\} \qquad (1)$$

Where, u and w are the axial and transverse displacements at the mid-plane, z is the perpendicular distance from the mid-plane to the layer plane, ϕ is the rotation of a plane after deformation, and t is the time. The strain-displacement relations are:

$$\left.\begin{aligned} \varepsilon_1 &= \frac{\partial U}{\partial x} = \frac{\partial u}{\partial x} + z\frac{\partial \phi}{\partial x} \\ \varepsilon_5 &= \frac{\partial W}{\partial x} + \frac{\partial U}{\partial z} = \frac{\partial w}{\partial x} + \phi \end{aligned}\right\} \quad (2)$$

Where the subscripts have the same meanings as those used in 3-D elasticity formulation, i.e. ε_1 is the axial or longitudinal strain, and ε_5 is the through-thickness shear strain. The stress-strain relationship of a lamina can be shown as:

$$\{\sigma_i\} = \left[\overline{C}_{ij}\right]\{\varepsilon_i\} \quad (3)$$

Where,

$$\left.\begin{aligned} \{\sigma\}^T &= \{\sigma_1 \quad \sigma_5\} \\ \left[\overline{C}_{ij}\right] &= \begin{bmatrix} \overline{C}_{11} & 0 \\ 0 & \overline{C}_{55} \end{bmatrix} \\ \{\varepsilon\}^t &= [\varepsilon_1 \quad \varepsilon_5] \end{aligned}\right\} \quad (4)$$

The elastic constants $\left[\overline{C}_{11}\right]$ and $\left[\overline{C}_{55}\right]$ for orthotropic beams can be expressed as:

$$\left.\begin{aligned} \overline{C}_{11} &= C_{11}.\cos^4\theta + C_{22}.\sin^4\theta + 2(C_{12} + 2.C_{66}).\sin^2\theta.\cos^2\theta \\ \overline{C}_{55} &= C_{44}.\sin^2\theta + C_{55}.\cos^2\theta \end{aligned}\right\} \quad (5)$$

Where;

$$C_{66} = G_{12};\ C_{44} = G_{23}; C_{55} = G_{13};\ C_{12} = \nu_{12}C_{22} = \nu_{21}C_{11} \quad (6)$$

Applying the energy approach for the beam element shown in Figure 2, the strain energy stored is given by:

$$U_s = \frac{1}{2}\int_e \{\varepsilon\}^T\{\sigma\}dV \quad (7)$$

Where, $dV = bdxdz$ and the subscript, e, means one element.

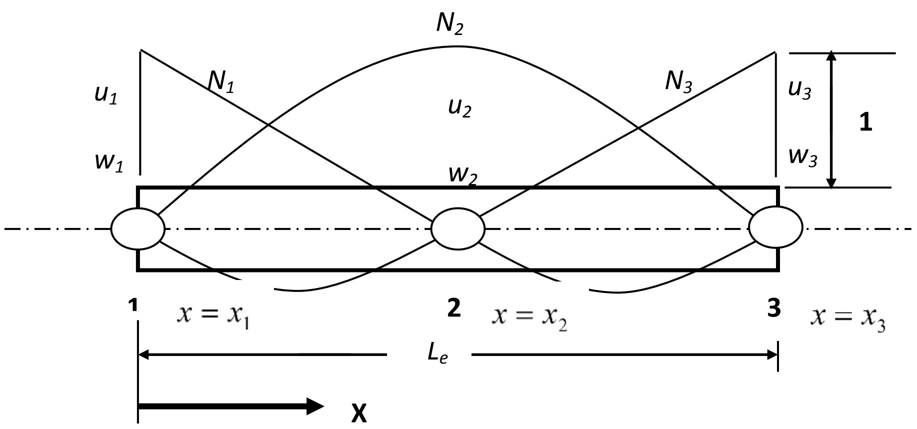


Figure 2 A 3-noded finite elements with nine degrees of freedom and shape functions.

Also, the kinetic energy is found as follows;

$$K.E. = \frac{1}{2} b\rho \int_e \left[\frac{\partial^2 W}{\partial t^2} W + \frac{\partial^2 U}{\partial t^2} U \right] dx\, dz \qquad (8)$$

The degrees of freedom at each node are; the axial displacement u, deflection w, and rotation ϕ, and can be written in terms of their nodal values as follows:

$$[u, w, \phi] = \sum_{i=1}^{i=3} [N_i u_i, N_i w_i, N_i \phi_i] \qquad (9)$$

Where, N_i is shape function and assumed as a second-order polynomial as:

$$N_i = a_i + b_i x + c_i x^2 \qquad (i = 1,2,3) \qquad (10)$$

The constants a_i, b_i, and c_i can be computed for each element from the following data:

$$N_i = \begin{cases} 1 \\ 0 \end{cases} \quad \text{at} \quad \begin{cases} x = x_i \\ x \neq x_i \end{cases} \qquad (i = 1,2,3) \qquad (11)$$

Eqns. (7), and (8) leads to the final form of the non-dimensional element stiffness and inertia matrices [K]e ,and [M]e respectively, which are given by:

$$[K]^e = \begin{bmatrix} Aq_1 & 0 & -Bq_1 & Aq_2 & 0 & -Bq_2 & Aq_3 & 0 & -Bq_3 \\ & A_s\lambda^2 q_1 & -A_s\lambda^2 q_{13} & 0 & A_s\lambda^2 q_2 & -A_s\lambda^2 q_{16} & 0 & A_s\lambda^2 q_3 & -A_s\lambda^2 q_{19} \\ & & Dq_1 + A_s\lambda^2 q_7 & -Bq_2 & -A_s\lambda^2 q_{14} & Dq_2 + A_s\lambda^2 q_8 & -Bq_3 & -A_s\lambda^2 q_{15} & Dq_3 + A_s\lambda^2 q_9 \\ & & & Aq_4 & 0 & -Bq_4 & Aq_5 & 0 & Bq_5 \\ & & & & A_s\lambda^2 q_4 & -A_s\lambda^2 q_{17} & 0 & A_s\lambda^2 q_5 & A_s\lambda^2 q_{20} \\ & & & & & Dq_4 + A_s\lambda^2 q_{10} & -Bq_5 & -A\lambda^2 q_{18} & D.q_5 + A_s\lambda^2 q_{11} \\ & & \text{Symmetric} & & & & Aq_6 & 0 & Bq_6 \\ & & & & & & & A_s\lambda^2 q_6 & A_s\lambda^2 q_{21} \\ & & & & & & & & Dq_6 + A_s\lambda^2 q_{12} \end{bmatrix} \qquad (12)$$

$$[M]^e = \frac{1}{\lambda^2} \begin{bmatrix} -q_7 & 0 & 0 & -q_8 & 0 & 0 & -q_9 & 0 & 0 \\ & -\lambda^2 q_7 & 0 & 0 & -\lambda^2 q_8 & 0 & 0 & -\lambda^2 q_9 & 0 \\ & & -q_7 & 0 & 0 & -q_8 & 0 & 0 & -q_9 \\ & & & -q_{10} & 0 & 0 & -q_{11} & 0 & 0 \\ & & & & -\lambda^2 q_{10} & 0 & 0 & -\lambda^2 q_{11} & 0 \\ & & & & & -q_{10} & 0 & 0 & -q_{11} \\ & & \text{Symmetric} & & & & -q_{12} & 0 & 0 \\ & & & & & & & 0 & -\lambda^2 q_{12} & 0 \\ & & & & & & & & -q_{12} \end{bmatrix} \qquad (13)$$

Where, λ is the aspect ratio (length to thickness ratio), and;

$$
\begin{aligned}
&q_1=\int_e \frac{\partial N_1}{\partial x}\frac{\partial N_1}{\partial x}dx && q_8=\int_e N_1N_2dx && q_{15}=\int_e N_1\frac{\partial N_3}{\partial x}dx\\
&q_2=\int_e \frac{\partial N_1}{\partial x}\frac{\partial N_2}{\partial x}dx && q_9=\int_e N_1N_3dx && q_{16}=\int_e N_2\frac{\partial N_1}{\partial x}dx\\
&q_3=\int_e \frac{\partial N_1}{\partial x}\frac{\partial N_3}{\partial x}dx && q_{10}=\int_e N_2N_2dx && q_{17}=\int_e N_2\frac{\partial N_2}{\partial x}dx\\
&q_4=\int_e \frac{\partial N_2}{\partial x}\frac{\partial N_2}{\partial x}dx && q_{11}=\int_e N_2N_3dx && q_{18}=\int_e N_2\frac{\partial N_3}{\partial x}dx\\
&q_5=\int_e \frac{\partial N_2}{\partial x}\frac{\partial N_3}{\partial x}dx && q_{12}=\int_e N_3N_3dx && q_{19}=\int_e N_3\frac{\partial N_1}{\partial x}dx\\
&q_6=\int_e \frac{\partial N_3}{\partial x}\frac{\partial N_3}{\partial x}dx && q_{13}=\int_e N_1\frac{\partial N_1}{\partial x}dx && q_{20}=\int_e N_3\frac{\partial N_2}{\partial x}dx\\
&q_7=\int_e N_1N_1dx && q_{14}=\int_e N_1\frac{\partial N_2}{\partial x}dx && q_{21}=\int_e N_3\frac{\partial N_3}{\partial x}dx
\end{aligned}
\qquad (14)
$$

For laminated beam with (n) layers, the constants in Eqn. (13) are:

$$
\left.
\begin{aligned}
&[A,B,D]=\sum_{k=1}^{k=n}\int_{Z_k}^{Z_{k+1}}[1,z,z^2]C_{11}^k bdz\\
&A_s=K_s\sum_{k=1}^{k=n}\int_{Z_k}^{Z_{k+1}}C_{55}^k bdz
\end{aligned}
\right\}
\qquad (15)
$$

The constants A, B, and D are the extensional, coupling, bending stiffnesses respectively, while the constant As is the shear coefficient, and Ks is the shear correction factor (taken $2/3$ or $5/6$).

The individual element stiffness and inertia matrices [K]e and [M]e must be linked together or assembled to characterize the unified behavior of the entire beam. Therefore, the global stiffness and inertia matrices are given respectively by,

$$
\left.
\begin{aligned}
&[K]=\sum_{n=1}^{N}[K]^e\\
&[M]=\sum_{n=1}^{N}[M]^e
\end{aligned}
\right\}
\qquad (16)
$$

Where, N is the total number of beam elements.

The beam end conditions which considered are; (1) Clamped-free beam (CF), (2) Hinged-hinged beam (HH), (3) Clamped-clamped beam (CC), (4) Hinged-clamped beam (HC), (5) Hinged-free beam (HF), and (6) Free-free beam (FF). Each beam has either movable ends or immovable ends. In the former group, the axial displacement at both beam-ends is considered, while neglected for the latter group.

The solution can be obtained after the incorporation of ends conditions which will modify both stiffness and inertia matrices. Thus, the non-dimensional natural frequencies can be determined from the relation:

$$
\left|[M]^{-1}[K]-\omega^2 I\right|=0 \qquad (17)
$$

Where, I is an identity matrix, and ω is the non-dimensional natural frequencies, which can be computed by computing the square root of the eigenvalues of the matrix $[M]^{-1}[K]$ using a suitable computer program (Here MATLAB was used).

3. NUMERICAL RESULTS

AS/3501-6 graphite-epoxy material was used for all numerical results because of its wide applications in modern industries. The mechanical properties of this material are tabulated in Table (1).

Table 1 Mechanical Properties of AS/3501-6 graphite-epoxy material

Property	Magnitude
E1	145 GN/m2
E2	9.6 GN/m2
G12	4.1 GN/m2
G13	4.1 GN/m2
G23	3.4 GN/m2
Poisson's ratio (ν)	0.3
Density (ρ)	1520 kg/m3

In order to check the validity of the present method, some comparisons were performed. These comparisons were selected to cover the cases of symmetrically and non-symmetrically laminated beams.

Table (2) shows comparisons with Chandrashekhara et al. [2]. The comparison shows the differences between the non- dimensional fundamental frequencies [$\bar{\omega}=\omega.L^2\sqrt{\rho/E_1h^2}$] of symmetric [$\theta/-\theta/-\theta/\theta$] angle-ply beams with different boundary conditions for aspect ratio of (L/h=15). The angle (θ) varies from 0^o to 90^o with a step (15^o). The comparison showed a difference of less than 0.06% for the angle zero, increasing to about 0.30% for higher values of the angle (θ).

Table 2 Non-dimensional fundamental frequencies [$\bar{\omega}=\omega.L^2\sqrt{\rho/E_1h^2}$] of symmetric [$\theta/-\theta/-\theta/\theta$] angle-ply beams (L/h = 15).

Beam Type \ θ (deg.)		0	15	30	45	60	75	90
Hinged-hinged	present	2.6545	2.5091	2.1021	1.5356	1.0108	0.7592	0.7303
	Ref. [2]	2.6560	2.5105	2.1032	1.5368	1.0124	0.7611	0.7320
Clamped-Clamped	Present	4.8397	4.6554	4.0927	3.1826	2.1996	1.6838	1.6225
	Ref. [2]	4.8487	4.6635	4.0981	3.1843	2.1984	1.6815	1.6200
Hinged-Free	present	4.0907	3.8707	3.2513	2.3825	1.5716	1.1814	1.1364
	Ref. [2]	4.0931	3.8728	3.2530	2.3841	1.5738	1.1840	1.1389
Free-free	Present	5.8895	5.5749	4.6875	3.4389	2.2702	1.7070	1.6420
	Ref. [2]	5.8923	5.5774	4.6894	3.4407	2.2730	1.7105	1.6453

Clamped-Free	present	0.9817	0.9246	0.7674	0.5547	0.3625	0.2715	0.2611
	Ref. [2]	0.9820	0.9249	0.7678	0.5551	0.3631	0.2723	0.2619
Clamped-Hinged	Present	3.7257	3.5550	3.0544	2.3017	1.5502	1.1745	1.1306
	Ref. [2]	3.7305	3.5593	3.0573	2.3032	1.5511	1.1753	1.1312

Table (3) presents a comparison with Abramovich and Livshits [4] of the non-dimensional frequencies of symmetric [0/90/90/0] cross-ply graphite-epoxy beams for aspect ratio of (L/h = 10). The beams considered are hinged-hinged, fixed-free, and fixed-fixed with immovable ends. Here, the authors introduced the secondary effect of coupling between bending and torsion in their analysis, which is small, compared with the other secondary effects. For the hinged-hinged beam, a percentage difference of less than 0.16% was recorded for the fundamental frequency, and less than 0.54% for both fixed-free and fixed-fixed beams. This difference was observed to increase as the mode order increases (less than 1.4%) for the seventh mode for all beams considered. In addition, Table 3 shows the modes with predominance of longitudinal vibration.

Table 3 Non-dimensional frequencies [$\overline{\omega} = \omega . L^2 \sqrt{\rho / E_1 h^2}$] of symmetric [0/ 90/ 90/ 0] cross-ply beams (L/h = 10)

Mode No.	Hinged-hinged (Immovable)		Fixed-free (Immovable)		Fixed-fixed (Immovable)	
	Present	Ref. [4]	Present	Ref. [4]	Present	Ref. [4]
1	2.3157	2.3194	0.8866	0.8819	3.6855	3.7576
2	6.9813	7.0029	4.1062	4.0259	7.7244	7.8718
3	12.004	12.037	8.9536	9.1085	12.381	12.573
4	17.010	17.015	11.504*	12.193*	17.192	17.373
5	22.015	21.907	13.924	14.080	22.119	22.200
6	23.007*	23.007*	18.980	18.980	23.007*	23.007*
7	27.094	27.094	24.037	24.037	27.125	27.125

(*) Modes with predominance of longitudinal vibration.

The last comparison was carried out with the results of Marur and Kant [6]. Table (4) compares the first six modes of the non-dimensional frequencies [$\overline{\omega} = \omega . L^2 \sqrt{\rho / E_1 h^2}$] of symmetric $[0/90/90/0]$ cross-ply, graphite-epoxy, clamped-free beam with aspect ratio of (L/h = 15). The authors applied the higher-order shear deformation theory (HOSDT) in the analysis, whereas, the present method uses first-order shear deformation theory. The comparison shows a percentage difference of less than 0.03% for the fundamental mode of vibration. This difference increases with the mode order to less than 3.6% for the sixth mode.

Table 4 Non-dimensional natural frequencies [$\overline{\omega} = \omega . L^2 \sqrt{\rho / E_1 h^2}$] of symmetric $[0/90/90/0]$ cross-ply clamped- free beam. (L/h = 15).

Mode No.	Present	Ref. [6]
1	0.9238	0.924

2	4.8886	4.985
3	11.4556	11.832
4	17.2550*	-
5	18.8481	19.573
6	26.7793	27.720

(*) Modes with predominance of longitudinal vibration.

It is obvious from the numerical results given in Table (2) above, and Table (5) below, that the values of non-dimensional natural frequencies of various beams generally decrease as the angle of orientation of fibers with respect to the longitudinal axis of the beam is increased.

Table 5 The first three non-dimensional modes of free vibration of symmetric $[\theta/-\theta/-\theta/\theta]$ laminated beams with immovable ends. L/h=10.

Angle (θ)	Mode No.	Beam type					
		CF	HH	CC	HC	HF	FF
30o	1	0.7465	1.9918	3.4380	2.7113	3.0503	4.3728
	2	3.7279	6.4128	7.4386	6.9645	7.9206	9.5329
	3	8.4193	11.4744	12.7816	11.7816	13.1707	14.9573
60o	1	0.3596	0.9943	2.0601	1.4899	1.5370	2.2088
	2	2.0893	3.6853	5.0523	4.3780	4.5879	5.5663
	3	5.3017	7.4807	8.8047	8.1620	8.6142	9.8029
90o	1	0.2597	0.7224	1.5544	1.1020	1.1184	1.6077
	2	1.5522	2.7525	3.9654	3.3519	3.4324	4.1694
	3	4.0677	5.7774	7.1377	6.4646	6.6580	7.5785

Similar values of frequencies for symmetric $[\theta/-\theta/-\theta/\theta]$ laminated beams with immovable ends and aspect ratio of 10 are plotted against the angle of orientation for the range from 0 up to 90 degrees in Fig. (3) to Fig. (8). The influence of fiber orientation becomes more noticeable as the mode order increases, and significant variations of frequencies were observed up to an angle of approximately 70 degrees. Beyond this angle, the variations in the frequencies are very small.

Increasing angle of orientation to more than 70 degrees leads to increase the coupling between bending and stretching effect, which causes the laminated beam to be stiffer, and thus the variation in natural frequencies decreases. In addition, the values of non-dimensional natural frequencies of the longitudinal modes of free vibration are observed to decrease as the angle of fibers orientation is increased.

4. CASE STUDIES

Some important cases, which related to the fiber orientation, were studied and presented here. The first case is repeated or alternating set of layers in similar beams. Table (6) shows the non-dimensional natural frequencies of symmetric $[60/-60/-60/60]$ and $[60/-60/-60/60]_2$ laminated beams respectively. Both beam laminations have the same aspect ratio of 15

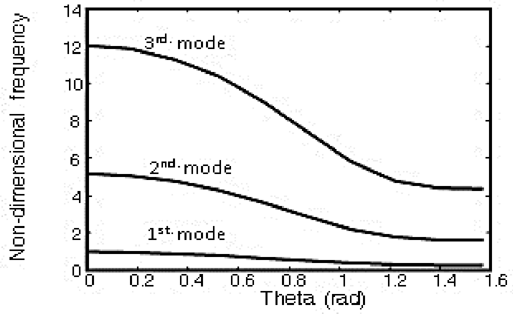


Figure 3 Clamped-free beam.

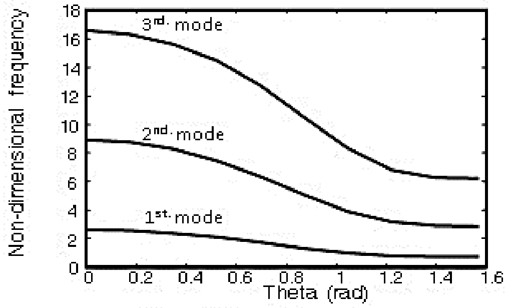


Figure 4 Hinged-hinged beam.

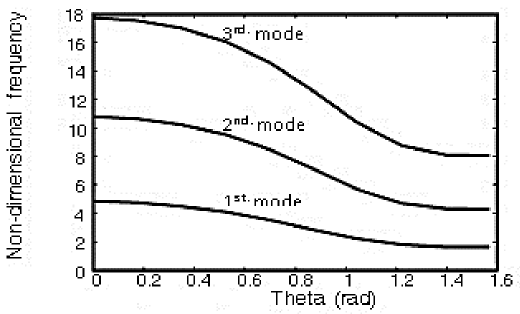


Figure 5 Clamped-clamped beam.

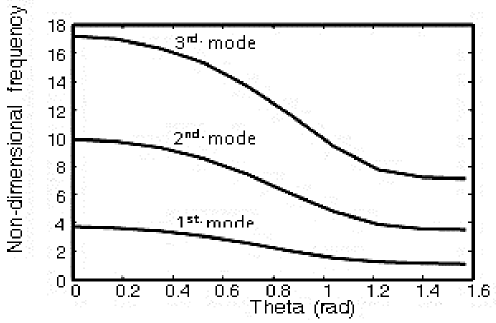


Figure 6 Hinged-clamped beam.

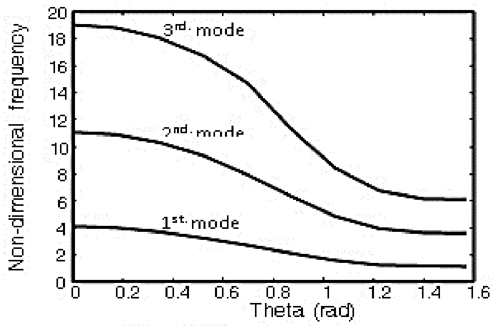


Figure 7 Hinged-free beam.

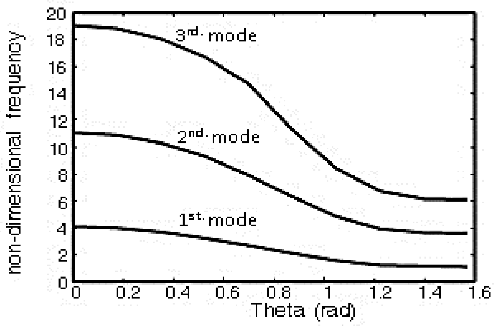
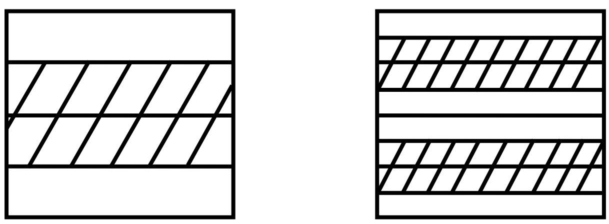


Figure 8 Hinged-free beam.

The first set of beams contains four layers oriented as shown in Fig. (9)a, whilst the other set contains eight layers as shown in Fig. (9) b. It is observed that these two sets have the same natural frequencies for same aspect ratio. In this case, equal values of the coefficients A, B, D, and A_s will be obtained for both beams and consequently the resulting inertia and stiffness matrices are the same for both.



(a) $[60/-60/-60/60]$ (b) $[60/-60/-60/60]_2$

Figure 9 Two sets of beam laminations

Table 6 Non-dimensional frequencies of a symmetric [60/-60/-60/60] and [60/-60/-60/60]2 laminated beams with immovable ends, (L/h = 15)

Mode No.	Beam type					
	CF	HH	CC	HC	HF	FF
1	0.3623	1.0101	2.1928	1.5475	1.5703	2.2680
2	2.1913	3.8941	5.6775	4.7703	4.8867	5.9775
3	5.8269	8.2922	10.3643	9.3341	8.4639	11.0406
4	8.4639	13.7890	15.8761	14.8504	9.6260	16.9278
5	10.6899	16.9278	16.9278	16.9278	15.3764	17.0272
6	16.4306	20.0246	21.9566	21.0111	21.7950	23.6152
7	22.7569	26.7316	28.4256	27.5969	25.3918	30.5717
8	25.3918	33.7270	33.8557	33.8557	28.6323	33.8557
9	29.4652	33.8557	35.1607	34.4580	35.7194	37.7414
10	36.4166	40.8914	42.0767	41.4943	42.3198	45.0240
11	42.3198	48.1496	49.1142	48.6389	42.9470	50.7839
12	43.5180	50.7839	50.7839	50.7839	50.2463	52.3564

The second case is reversed lamination order of similar beams. Two lamination sets were considered which are $[30/60/90]$ and $[90/60/30]$. The results of their non-dimensional natural frequencies are given in Table (7) for an aspect ratio of 15. It is obvious that these frequencies are the same for both sets.

Table 7 Non-dimensional frequencies $\left[\bar{\omega} = \omega\sqrt{\rho L^4 / E_1 h^2}\right]$ of a non-symmetric [30/60/90] and [90/60/30] laminated beams with immovable ends, (L/h = 15).

Mode No.	Beam type					
	CF	HH	CC	HC	HF	FF
1	0.4110	1.4577	2.4557	1.8383	1.7642	3.121
2	2.4597	4.3308	6.2650	5.3717	5.4235	6.5715
3	6.4537	9.4175	11.2789	10.2659	9.9128	12.7081
4	11.5603	14.6227	17.0649	15.9794	12.1937	16.8666
5	12.2368	21.6173	23.3665	21.8096	16.7618	22.6558
6	17.7284	21.9998	23.9363	23.6494	23.5247	25.6082
7	24.2916	29.1040	30.0143	29.4584	29.9290	33.7244
8	31.0676	35.2855	36.8563	36.2011	34.6973	38.3378
9	35.5049	42.9879	43.8799	43.1304	37.6734	44.3434
10	38.3008	43.6059	46.0794	45.0979	45.2069	47.7745

11	45.4855	50.7299	51.0365	50.8438	51.5700	55.9195
12	52.3938	57.2216	58.1143	57.7467	55.9437	60.1605

The last case is the incorrectly oriented layer. The cases studied here are symmetric $\left[0/0/0/0/0/0/0/0\right]$ laminated beams with immovable ends. If the orientation of first layer is incorrectly made 90o rather than 0o, the values of non-dimensional frequencies generally decrease as could be seen in Tables (8) and (9) respectively.

Table 8 Non-dimensional frequencies $\left[\bar{\omega} = \omega\sqrt{\rho L^4 / E_1 h^2}\right]$ of a symmetric [0/0/0/0/0/0/0/0] laminated beam with immovable ends, (L/h = 15).

Mode No.	Beam type					
	CF	HH	CC	HC	HF	FF
1	0.9817	2.6541	4.8376	3.7246	4.0898	5.8874
2	5.1648	8.9441	10.7896	9.9260	11.1075	13.4514
3	12.0255	16.5385	17.7513	17.1675	19.0527	21.7104
4	19.5924	24.3920	25.1158	24.7637	23.6325	29.8793
5	23.6325	32.2089	32.6494	32.4317	27.0890	37.8995
6	27.4086	39.9335	40.2085	40.0724	35.0157	45.7461
7	35.2037	47.2649	47.2649	47.2649	42.8138	47.2649
8	42.9341	47.5714	47.7513	47.6616	50.5041	53.4823
9	50.5808	55.1405	55.2611	55.2012	58.1111	61.1084
10	58.1648	62.6584	62.7423	62.7003	65.6557	68.6789
11	65.6912	70.1399	70.1988	70.1695	70.8974	76.1718
12	70.8974	77.5974	77.6400	77.6186	73.1538	83.6474

This occurs because increasing of angle of orientation cause a decrease in the values of the natural frequencies as discussed above. The maximum and minimum percentage decreases in the values of the fundamental mode are about 16% and 11% respectively. This percentage increases if the mode order is increased. It may be expected that these differences decrease as the number of layers is increased.

Table 9 Non-dimensional frequencies $\left[\bar{\omega} = \omega\sqrt{\rho L^4 / E_1 h^2}\right]$ of a non-symmetric [90/0/0/0/0/0/0/0] laminated beam with immovable ends, (L/h = 15)

Mode No.	Beam type					
	CF	HH	CC	HC	HF	FF
1	0.8264	2.2976	4.3185	3.2598	3.4852	5.1003
2	4.5310	7.8994	9.9409	8.9775	9.8368	11.9430
3	10.8769	15.1199	16.6260	15.8984	17.4000	19.9305
4	18.1346	22.7868	23.8005	23.3130	22.1084	27.9180
5	22.1984	30.5716	31.2166	30.8959	25.3333	35.9901

6	25.7873	38.2588	38.7125	38.4945	33.2368	42.8836
7	33.5147	43.9545	44.2959	44.1090	41.0260	44.8354
8	41.2171	45.9369	46.2202	46.0926	48.7384	51.5969
9	48.8585	53.5260	53.7075	53.6152	56.3351	59.3045
10	56.4303	61.0294	61.1672	61.0995	63.8921	66.7923
11	63.9352	68.5084	68.6022	68.5547	65.9928	74.3334
12	66.2190	75.9420	76.0163	75.9798	71.4242	81.6580

6. CONCLUSIONS

In this paper, a first-order shear deformation theory was applied in the analysis, and a finite element model has been formulated to predict the non-dimensional natural frequencies and to study the influence of fibers orientation on the natural frequencies. Twelve end conditions were studied which are clamped-free, hinged-hinged, clamped-clamped, hinged-clamped, hinged-free, and free-free beams with immovable and movable ends. The main conclusions are:

(1) Similar beams, which are either symmetrically laminated $[\theta/-\theta/-\theta/\theta]$ or anti-symmetrically laminated $[\theta/-\theta/\theta/-\theta]$, have equal natural frequencies, since the coefficients $\overline{C}_{11}$ and $\overline{C}_{55}$ are equal for both cases (see Eqn. (5)).

(2) Repetition of a set of layers in symmetric or anti-symmetric similar beams does not alter the natural frequencies of the beam. e.g. $[\theta/-\theta/-\theta/\theta]$ or $[(\theta/-\theta/-\theta/\theta)_n]_S$, where $n = 1, 2, 3, \ldots$.

(3) If the lamination order of a laminated beam is reversed, the natural frequencies will remain the same. e.g. (30/60/90) or (90/60/30).

(4) The natural frequencies of a laminated beam generally decrease as the fiber orientation angle increases.

(5) The error in natural frequencies of a laminated beam resulting from an incorrectly oriented layer will decrease if the number of layers is increased and/or the angle of orientation of the incorrectly oriented layer is decreased.

ACKNOWLEDGEMENT

The authors would like to acknowledge with deep thanks and profound gratitude Mr. Osama Mahmoud of Daniya Center for Publishing and Printing Services, Atbara, who spent many hours in editing, re – editing of the manuscript in compliance with the standard format of International Journal of Engineering Research and Advanced Technology (IJERAT).

REFERENCES

[1] Teoh L.S. and Huang C.C. '*The vibration of beams of fiber reinforced material*', (1976), Journal of sound and vibration 51(4), pp. (467-473).

[2] Chandrashekhara K., Krishnamurthy K. and Roy S. ' *Free vibration of composite beams including rotary inertia and shear deformation*', (1990), Composite Structure 14, pp. (269-279).

[3] Abramovich H. '*Shear deformation and rotary inertia effects of vibrating composite beams*', (1992), Composite Structure 20, pp. (165-173).

[4] Abramovich H. and Livshits A. ' *Free vibrations of non-symmetric cross-ply laminated composite beams*', (1994), Journal of sound and vibration 176(5), pp. (597-612).

[5] Marur S.R. and Kant T. ' *Transient dynamics of laminated beams: an evaluation with a higher-order refined theory*', (1998), Composite Structure 41, pp.(1-11).

[6] Marur S.R. and Kant T. '*Free vibration analysis of fiber-reinforced composite beams using higher-order theories and finite element modeling*', (1996), Journal of sound and vibration 194, pp. (337-351).

[7] McCarthy T.R. and Chattopadhyay A. ' *Investigation of composite box beam dynamics using a higher-order theory*', (1998), Composite Structure 41, pp. (273-284).

[8] Teboub Y. and Hajela P. ' *Free vibration of generally layered composite beams using symbolic computations*', (1995), Composite Structure 33, pp. (123-134).

[9] Hodges D.H., Atilgan A.R., Fulton M.V., and Rehfield L.W. ' *Free vibration analysis of composite beams*', (1991), Journal of American Helicopter Society 36, pp. (36-47).

[10] Banerjee J.R. '*Explicit analytical expression for frequency equation and mode shapes of composite beams*', (2001), International Journal of Solid and Structures 38, pp.(2415-2426).

[11] Kadivar M.H. and Mohebpour S.R. ' *Finite element dynamic analysis of unsymmetric composite laminated beams with shear effect and rotary inertia under the action of moving loads*', (1998), Finite Elements in Analysis and Design 29, pp.(259-273).

[12] Shimpi R.P. and Ainapure A.V. ' *A beam finite element based on layerwise trigonometric shear deformation theory*', (2001), Composite Structures 53, pp.(153-162).

[13] N. D. Phan and J. N. Reddy, 'Analysis of laminated composite plate using higher order shear deformation theory', International Journal of Numerical Method in Engineering, Vol. 21, (1985), PP. (2201 – 2219).

[14] Mahmoud Yassin Osman and Osama Mohammed Elmardi Suleiman, 'Free vibration analysis of laminated composite beams using finite element method', International Journal of Engineering Research and Advanced Technology (IJERAT), Vol. 03, Issue 2, (2017), PP. (5 – 22).

[15] Mahmoud Yassin Osman and Osama Mohammed Elmardi Suleiman, 'Free vibration of laminated plates', International Journal of Engineering Research and Advanced Technology (IJERAT), Vol. 03, Issue 4, (2017), PP. (31 – 47).